FRIDAY, JUNE 5, 2009

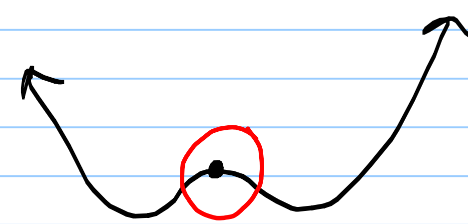
LIMITS

CONTINUITY $\longrightarrow$ $\lim_{x \to a} f(x) = f(a)$

DERIVATIVES —

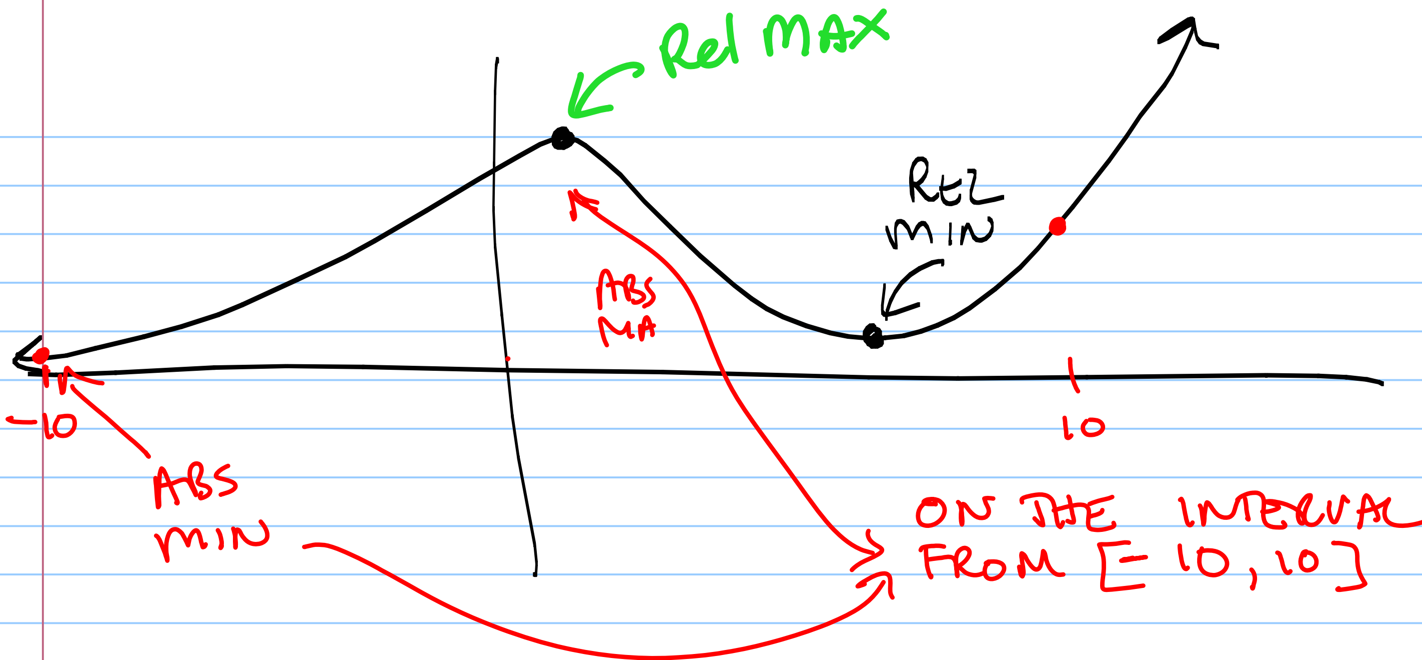
MAXIMA / MINIMA — EXTREMA

RELATIVE EXTREMA
LOCAL



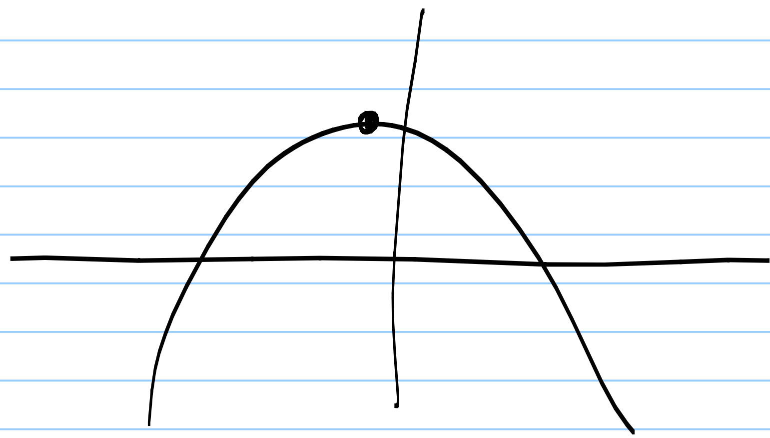
ABSOLUTE EXTREMA

AN ABSOLUTE MAXIMUM OF $f(x)$ ON SOME SET OF REAL #'S IS THE ABS MAXIMUM VALUE THE FUNCTION ATTAINS



NO ABS MAX

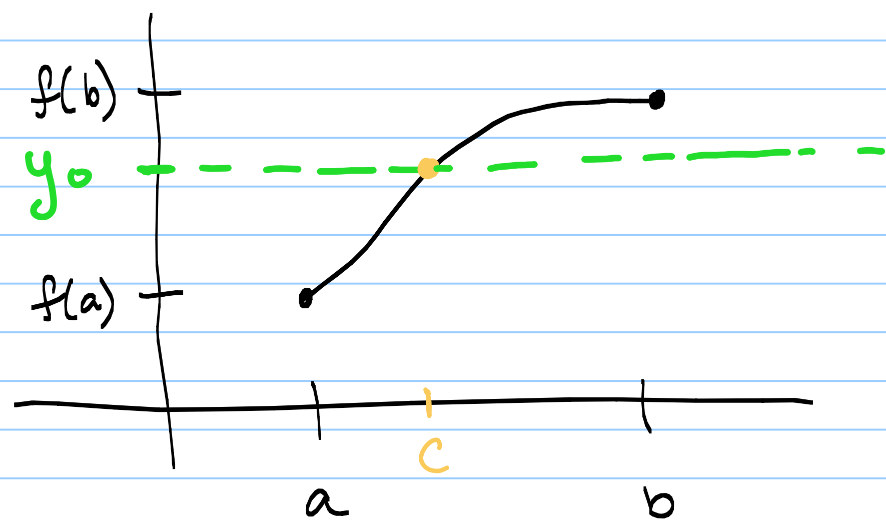
NO ABS MIN



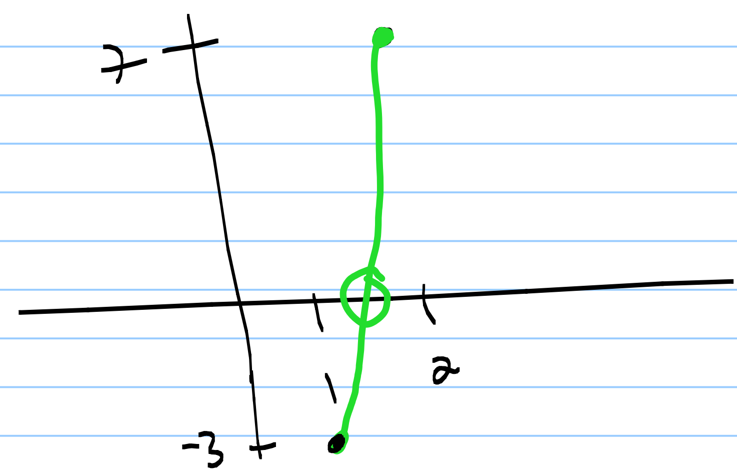
FOR A CONTINUOUS FUNCTION, $f(x)$, ON ANY CLOSED INTERVAL, THERE WILL ALWAYS BE ABS EXTREMA, AND THE WILL BE AT CRITICAL POINTS, OR AT ONE OF THE END POINTS.

THE INTERMEDIATE VALUE THEOREM

IF $f(x)$ IS CONTINUOUS ON $[a, b]$ AND $f(a) \neq f(b)$, THEN FOR ANY y-VALUE, y_0, BETWEEN $f(a)$ AND $f(b)$, THERE IS AN x-VALUE, c, BETWEEN a and b SO THAT $f(c) = y_0$.



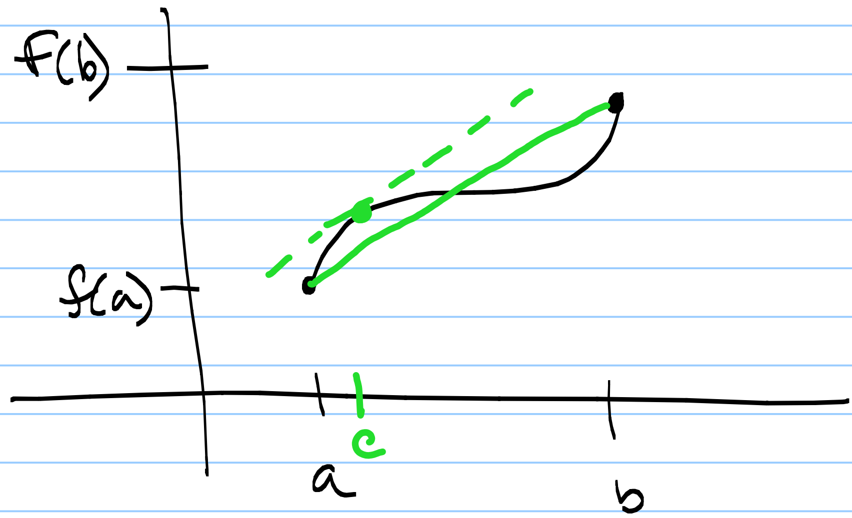
IF $y=f(x)$, AND YOU KNOW THAT $f(x)$ IS CONTINUOUS, AND YOU KNOW THAT $f(1)=-3$ AND $f(2)=7$, THEN BY THE I.V.T., $f(x)=0$ HAS A SOLUTION BETWEEN 1 AND 2.



THE MEAN VALUE THEOREM

IF $f(x)$ IS DIFFERENTIABLE ($\Rightarrow$ CONTINUOUS) ON $[a,b]$ THEN THERE IS A c, BETWEEN a & b SO THAT

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



L'Hôpital's Rule

IF $\lim_{x \to a} \frac{f(x)}{g(x)}$ AND $\lim_{x \to a} f(x) = \overset{\infty}{0} = \lim_{x \to a} g(x)$, THEN

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

$$\lim_{x\to\infty} \frac{x}{x} \qquad \lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$

INDETERMINATE FORMS

$$\frac{0}{0},\ \frac{\infty}{\infty};\ \infty-\infty;\ \underbrace{1^{\infty},\ 1^{0},\ \infty^{0}}_{\text{ex } \lim_{x\to 0} (\cos x)^{\frac{1}{x}}}$$

$\infty-\infty$: FIND COMMON DENOM.

TRICK: FIND LIMIT OF $\ln(*)$

$$\lim_{x\to 0} (\cos x)^{1/x} = e^0 = 1$$

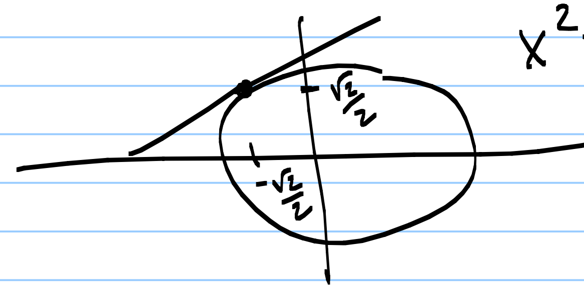
$$\lim_{x\to 0} \ln\left((\cos x)^{1/x}\right) = \lim_{x\to 0} \frac{\ln \cos x}{x} = \lim_{x\to 0} \frac{\frac{1}{\cos x}\cdot(-\sin x)}{1}$$

$$= \lim_{x\to 0} \frac{-\sin x}{\cos x} = 0$$

$$\boxed{e^{\ln\left((\cos x)^{1/x}\right)} = (\cos x)^{1/x}}$$

IMPLICIT DIFFERENTIATION

y' IS STILL THE SLOPE OF THE TANGENT LINE!



$x^2+y^2=1$

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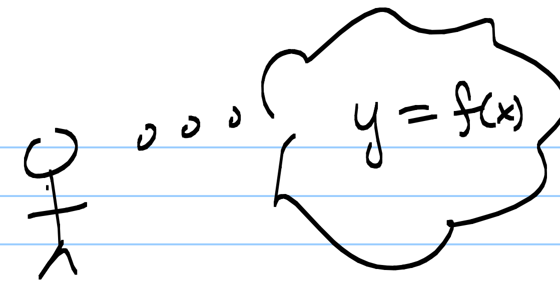


$$2x + 2yy' = 0$$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

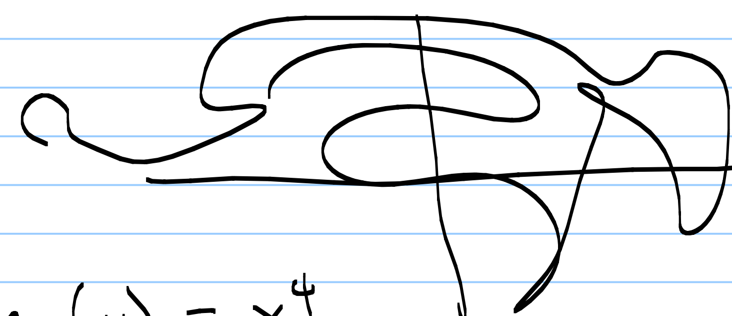
$$(f(x))^2 \xrightarrow[\frac{d}{dx}]{} 2 \cdot f(x) \cdot f'(x)$$

$$y^2 \qquad 2yy'$$

EQN OF LINE ABOVE:

$$m = -\frac{\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}} = +1 \quad , \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$y - \frac{\sqrt{2}}{2} = 1\left(x + \frac{\sqrt{2}}{2}\right)$$



<u>ex</u> $\ln(x \cdot y) - e^{y^2 - 3x} + \tan(y) = x^4$

$$\frac{1}{xy}\cdot\left(1\cdot y + y'x\right) - e^{y^2-3x}\cdot\left(2yy'-3\right) + \sec^2(y)\cdot y' = 4x^3$$

$$\frac{1}{x} + \frac{y'}{y} - 2yy'e^{y^2-3x} + 3e^{y^2-3x} + y'\sec^2 y = 4x^3$$

$$\frac{y'}{y} - 2yy'e^{y^2-3x} + y'\sec^2 y = 4x^3 - \frac{1}{x} - 3e^{y^2-3x}$$

$$y'\left(\frac{1}{y} - 2ye^{y^2-3x} + \sec^2 y\right) = 4x^3 - \frac{1}{x} - 3e^{y^2-3x}$$

$$y' = \frac{4x^3 - \frac{1}{x} - 3e^{y^2-3x}}{\frac{1}{y} - 2ye^{y^2-3x} + \sec^2 y}$$

LOGARITHMIC DIFFERENTIATION

$$y = x^x$$

$$\ln y = \ln x^x = x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$\frac{y'}{y} = \ln x + 1$$

$$y' = \overset{x^x}{y}(\ln x + 1)$$

$$y' = x^x(\ln x + 1)$$

$$y = (x\text{'s})^{(x\text{'s})}$$

↑ MUST USE L.D.

$$y = \sqrt{\frac{(x-1)^3 (x^2+4)^8}{7(x^7+9x)^2}} = \left(\frac{(x-1)^3 (x^2+4)^8}{7(x^7+9x)^2} \right)^{1/2}$$

$$\ln y = \frac{1}{2} \ln \left(\frac{(x-1)^3 (x^2+4)^8}{7(x^7+9x)^2} \right)$$

$$\ln y = \frac{1}{2} \left[\ln\left((x-1)^3\right) + \ln\left((x^2+4)^8\right) - \ln 7 - \ln\left((x^7+9x)^2\right) \right]$$

$$\ln y = \frac{1}{2} \left[3 \ln (x-1) + 8 \ln (x^2+4) - \ln 7 - 2 \ln (x^7+9x) \right]$$

$$\frac{1}{y} \cdot y' = \frac{1}{2} \left[3 \frac{1}{x-1} + 8 \cdot \frac{1}{x^2+4} \cdot 2x - 0 - 2 \frac{1}{x^7+9x} (7x^6+9) \right]$$

$$y' = \frac{1}{2} \left[\frac{3}{x-1} + \frac{16x}{x^2+4} - \frac{2(7x^6+9)}{x^7+9x} \right] \cdot y \quad \text{, where } y \text{ is as above.}$$

$s(t) \leftarrow$ position function

$v(t) = s'(t) \leftarrow$ velocity function

$a(t) = v'(t) = s''(t) \leftarrow$ acceleration

$j(t) = a'(t) = v''(t) = s'''(t) \leftarrow$ jerk

CALC I PROBLEMS

#2

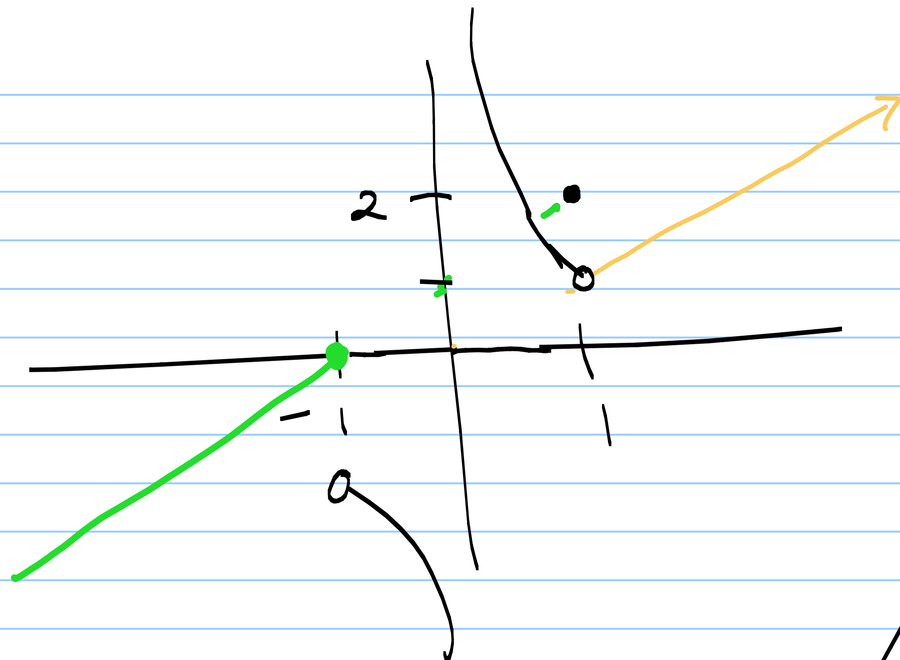
$$y = \begin{cases} x+1 & x \le -1 \\ 1/x & -1 < x < 1 \\ 2 & x = 1 \\ x & x > 1 \end{cases}$$

Disconitity

AT 0 – infinte

AT 1 – removable

AT –1 – jump



$x \overset{?}{=} 1$ $\qquad$ $x \overset{?}{=} -1$

#4 a) $\frac{1}{3}(2x^5 - x)^{-2/3}(10x^4 - 1)$

b) 5

c) $\ln 7 \cdot 7^{-t} \cdot (-1)$

d) $\frac{1}{2} x^{-1/2}$

e) $-8\sin(2\theta)$

f) $\sec^2(\alpha)$.

5) $y' = 2\cos 2x - 2x\sin x^2 - 6x\sec^2(3x^2-1)$

$\left(\sec(3x^2-1)\right)^2$

6) $y' = \dfrac{1}{e^x + e^{x^2}}\left(e^x + 2xe^{x^2}\right)$

7) $y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

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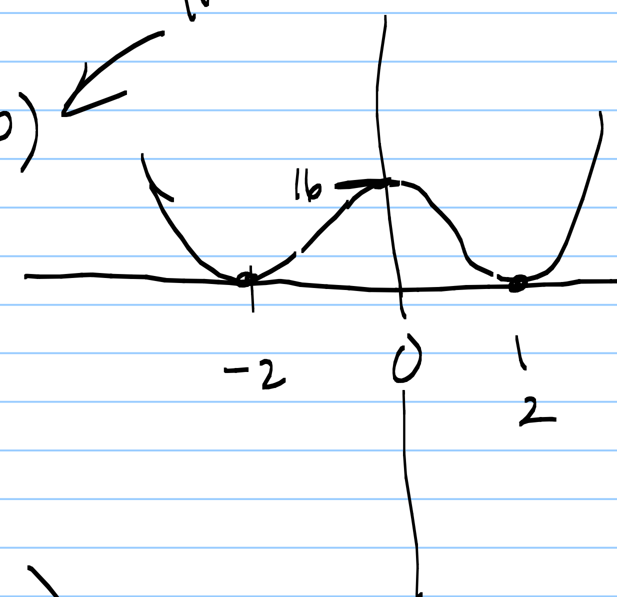
9) a) $-2, 0, 2$

$(-2,0) \cup (2,\infty)$ increasing

b) y': $-$ | -2 | $+$ | 0 | $-$ | 2 | $+$

c) $(-2,0)$ $(0,16)$ $(2,0)$

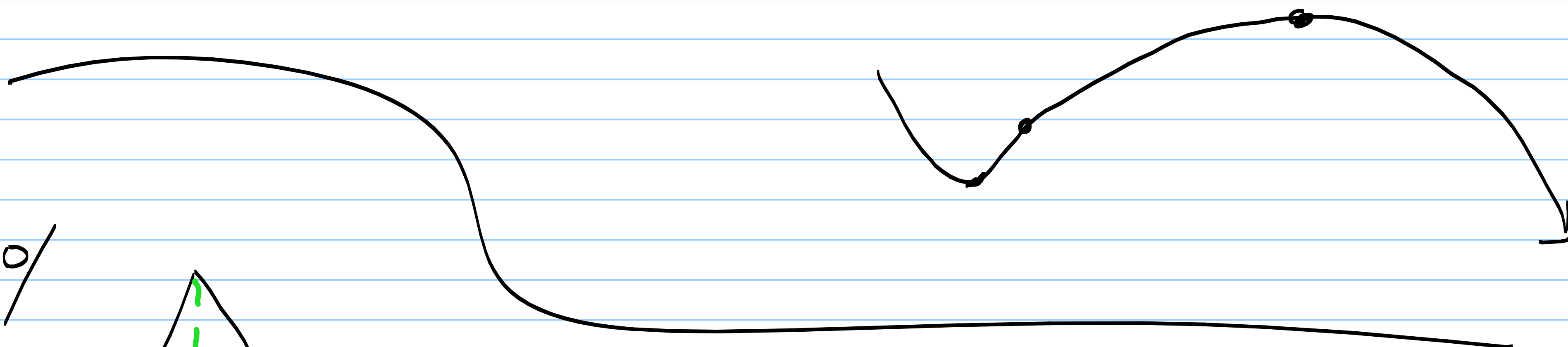


$+ \quad - \quad +$

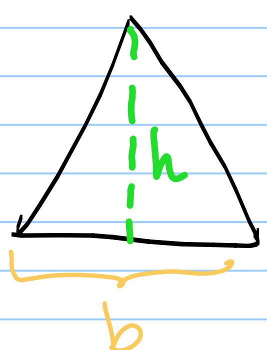
y'': $-\frac{2}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$

PoI

$\left(-\frac{2}{\sqrt{3}}, \left(\frac{4}{3}-2\right)^2\right)$



10/



$\frac{dh}{dt} = 1\ cm/min$

$\frac{dA}{dt} = 2\ cm^2/min$

WHAT IS $\frac{db}{dt}$ WHEN $h=10$ AND $A=100$?

$$A = \frac{1}{2} b \cdot h$$

$100 = \frac{1}{2} b \cdot 10$

$20 = b$

$$\underset{2}{\frac{dA}{dt}} = \frac{1}{2}\left(\frac{db}{dt} \cdot h + \underset{1}{\frac{dh}{dt}} \cdot b\right)$$

?

$$2 = \frac{1}{2}\left(\frac{db}{dt}\, \underset{10}{h} + \underset{20}{b}\right)$$

$$2 = \frac{1}{2}\left(\frac{db}{dt} \cdot 10 + 20\right)$$

$$\frac{4 - 20}{10} = \frac{db}{dt}$$

$$-1.6 \text{ cm/min} = db/dt$$