JUNE 1, PM

Note Title 6/1/2009

- MODULAR ARITHMETIC

$a \bmod b = r$

$12 \bmod 3 = 0$

$12 \bmod 5 = 2$

$3 \bmod 12 = 3$

$(5 = 8) \bmod 3$

$5 \bmod 3 = 8 \bmod 3$

$2 = 2$

- GROUP IS A SET, G, EQUIPPED W/ A BINARY OPERATION: $*$
- G IS CLOSED UNDER $*$
- THERE IS AN IDENTITY ELEMENT, e,

$$e * g = g * e = g$$

- $*$ IS ASSOCIATIVE
- IF $g \in G$ THEN THERE IS AN g^{-1} SO THE

$$g * g^{-1} = g^{-1} * g = e$$

<u>ex</u> $\mathbb{Z}$; $+$ IS A GROUP.

<u>ex</u> POSITIVE RATIONAL NUMBERS, $\mathbb{Q}^{>0}$; $\cdot$ ← MULTIPLICATION

<u>ex</u> $\{ f \mid f$ is a 1-1 continuous function on $\mathbb{R} \}$

WITH COMPOSITION, $\circ$, AS OPERATION.

Polynomial Long Division

$$\frac{93}{7} = 13 + \frac{2}{7}$$

$$\begin{array}{r} 13 \\ 7\overline{)\,93} \\ \underline{7} \\ 23 \\ \underline{21} \\ 2 \end{array} \leftarrow \text{Remainder}$$

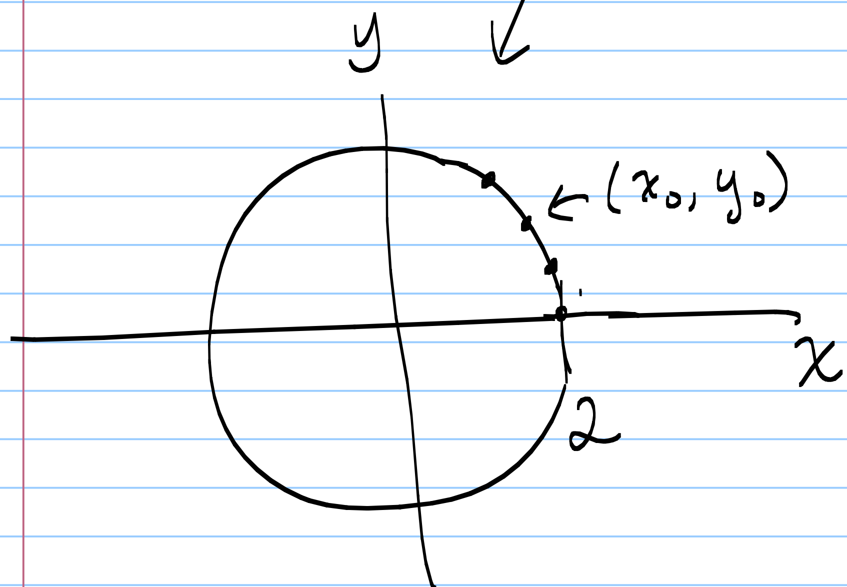
$$\frac{5x^3 - 7x^2 + 8}{x - 2}$$

$$= 5x^2 + 3x + 6 + \frac{20}{x-2}$$

$$\begin{array}{r} 5x^2 + 3x + 6 \\ x-2\overline{)\,5x^3 - 7x^2 + 0x + 8} \\ \underline{-(5x^3 - 10x^2)} \\ 3x^2 + 0x \\ \underline{-(3x^2 - 6x)} \\ 6x + 8 \\ -(6x - 12) \end{array}$$

x-y EQUATIONS

$$x^2+y^2=4$$



$$y=mx+b$$

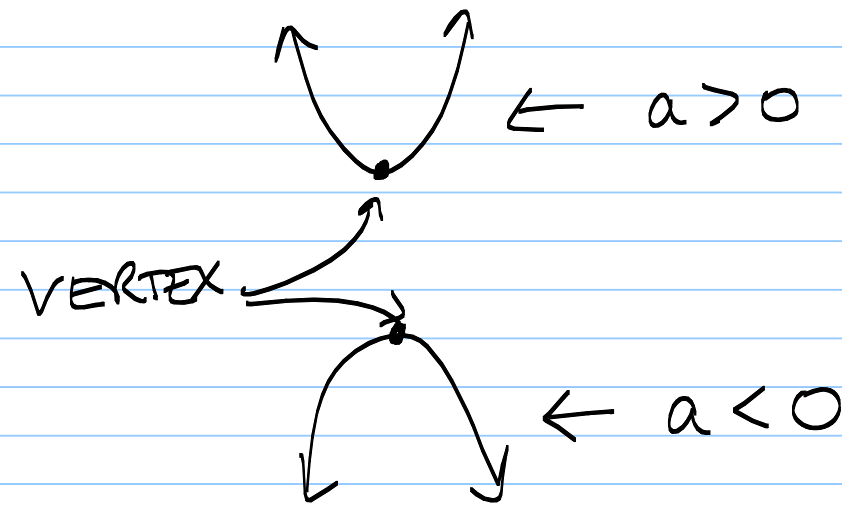
$$y-y_0=m(x-x_0)$$

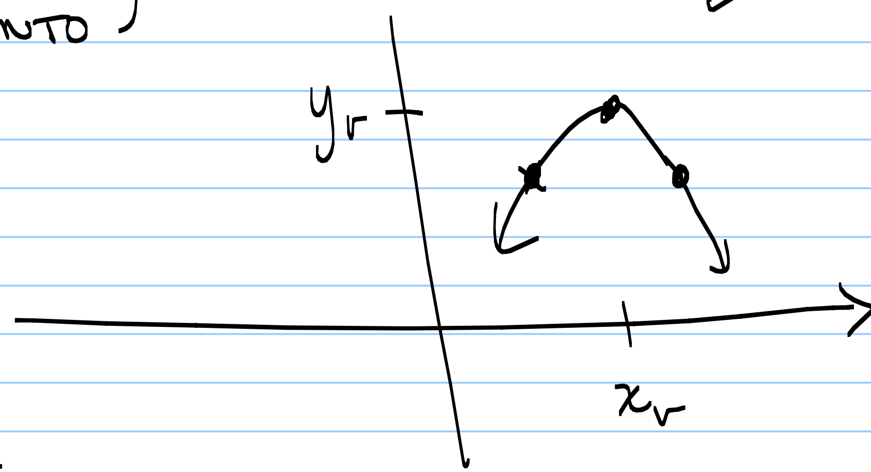
QUADRATIC EQUATIONS

$$y = ax^2 + bx + c$$

$$x_v = \frac{-b}{2a}$$

$y_v =$ PLUG INTO





$$y - k = a(x - h)^2$$

$\hookrightarrow$ (h, k) IS THE VERTEX

$$y = x^2 - 6x + 10$$

COMPLETE THE SQUARE

1. $\frac{1}{2}$ coeff of x & SQUARE IT.
2. ADD & SUBTRACT IT.

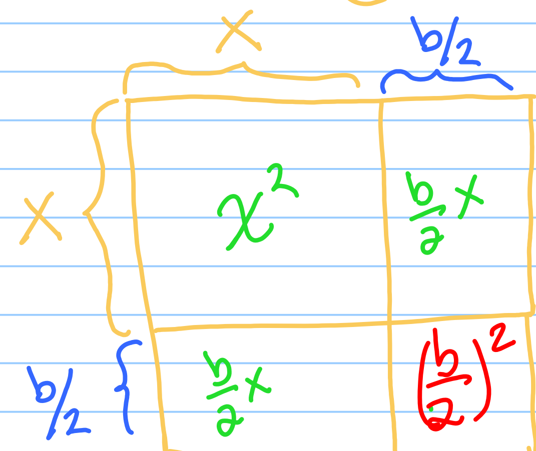
$$y - 10 = x^2 - 6x + 9 - 9$$

$$y - 10 = (x-3)^2 - 9$$

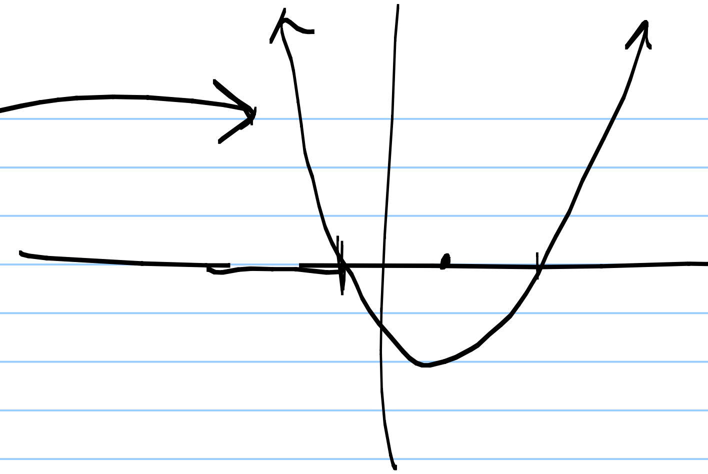
$$y - 1 = (x-3)^2$$

COMPLETE THE SQUARE:

$$\underbrace{x^2 + bx}_{} + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$



$$y = ax^2 + bx + c$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#4 $y + 5 > x^2 + 4x$

$y > x^2 + 4x - 5$

$y = x^2 + 4x - 5$

$x_v = -2$

$y_v = -9$

#6

PIECEWISE FUNCTION

$$P = \begin{cases} 8 & x \le 6 \\ 8 + 0.75(x-6) & x > 6 \end{cases}$$

ceiling $\lceil x \rceil \leftarrow$ ROUND UP.

floor $\lfloor x \rfloor \leftarrow$ Round down

MC
#8

$\cancel{(x+5)(9x-7)}$ $\cancel{(x+7)(9x-5)}$

$\cancel{(3x-7)(3x+5)}$ $(3x-5)(3x+7)$

$$9x^2 + 6x - 35 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 + 4 \cdot 9 \cdot 35}}{18} = \frac{-6 \pm 36}{18}$$

$$x = \frac{30}{18}, \quad \frac{-42}{18} \quad \cdots \quad \frac{5}{3}, \quad \frac{-7}{3}$$

$$x = \frac{5}{3} \qquad x = \frac{-7}{3}$$

$$3x = 5 \qquad 3x = -7$$

$$3x - 5 = 0 \qquad 3x + 7 = 0$$

$$(3x-5)(3x+7)$$

13

33

Power			1's PLACE
0	4	8	1
1	5	9	3
2	6	10	9
3	7	11	7

PROOF

AXIOMS $\xrightarrow[\text{RULES OF INFERENCE}]{\text{PROOF}}$ THEOREM

MODUS PONENS
RULE OF DETACHMENT

IF YOU KNOW P, AND
YOU KNOW THAT $P \to Q$,

THEN YOU CAN CONCLUDE
Q IS TRUE.

IF "x AND y ARE ODD" AND
"IF x & y are odd THEN x+y IS EVEN"
THEN "x+y is even"

THEOREMS IF <u>A</u> THEN <u>C</u>.

DIFFERENT KINDS OF PROOFS

- DIRECT PROOF

  A → ___ → ___ → ___ → <u>C</u>

- CONTRAPOSITIVE (USE IF (NOT C) THEN (NOT A)).

  NOT C → ___ → ___ → ___ → NOT A

- CONTRADICTION

  (A & NOT C) → ___ → ___ → ___ → CRAZY.

- INDUCTION : PROVE ALL NATURALS NUMBERS HAVE SOME PROPERTY.

1. PROVE THAT 0 HAS THE PROPERTY.

2. IF n HAS THE PROPERTY, THEN $n+1$ HAS IT.

DIRECT PROOFS

$2 \mid 6$ ✓

$2 \nmid 7$

THM IF $a \mid b$ AND $b \mid c$ THEN $a \mid c$.

PROOF ASSUME $a \mid b$ AND $b \mid c$.

THEN $b = na$ AND $c = mb$.

SUBSTITUTION YIELDS $c = m(na)$.

SO $c = (mn)a$, AND mn MUST BE A NATURAL NUMBER SINCE m AND n ARE. SO, WE CONCLUDE THAT $a \mid c$.

IF a IS ODD, THEN $a = b^2 - c^2$.

THM EVERY ODD INTEGER IS THE DIFFERENCE OF 2 PERFECT SQUARES.

PROOF ASSUME a is odd.

THEN $a = 2k + 1$ FOR SOME NUMBER k.

ADDING "0" WRITTEN AS $k^2 - k^2$, WE HAVE

$$a = 2k + 1 + k^2 - k^2.$$

SO $a = \underbrace{k^2 + 2k + 1} - k^2$

OR $a = (k+1)^2 - k^2$, SO a IS

THE DIFFERENCE OF TWO PERFECT SQUARES.