PROJECT: CONFIDENCE

ALGORITHMIC LEARNING THEORY, SUMMER 2014

1. Introduction

A learner identifies a language if he or she is correct about her conjecture in the limit, but we don't make any requirements on what happens on texts for languages that are not in the class to be learned. Imagine a very bold child that is sure he can always figure out the language being presented: No matter what text he's given, eventually he'll make a conjecture about the grammar and stick to it. We call such learners *confident* (perhaps we should call them overconfident) and in your project you'll investigate the consequences of this confidence.

2. Computability theory: Lemmas and exercises

Exercise 1. $\overline{\mathbb{K}}$ *is not computably enumerable.*

Exercise 2. *Let* $L_x = \mathbb{K} \cup \{x\}$*. Find a function* $h : \mathbb{N} \to \mathbb{N}$ *so that for each* x*,* $h(x)$ *is an index for* L_x *(i.e., so that* $W_{h(x)} = L_x$*).*

3. Learning Theory I: Identification, lemmas and exercises

Lemma 1. *Let* $\mathcal{L} = \{L_x \mid x \in \overline{\mathbb{K}}\}$*. Then* $\mathcal{L}$ *is identifiable. Moreover,* $\mathcal{L}$ *is recursively identifiable.*

Lemma 2. *If* $\varphi \in \mathcal{F}$ *identifies* RE_{FIN} *then there exists a text for* $\mathbb{N}$ *on which* φ *does not converge.*

4. Learning Theory II: Limitations

Definition 1. *A learner* $\varphi \in \mathcal{F}$ *is called* confident *if for every text* $t \in \mathcal{T}$*, there are only finitely many* n *for which* $\varphi(t \upharpoonright n) \neq \varphi(t \upharpoonright (n+1))$*. In other words,* φ *converges to a value on every text.*

Proposition 1. $\left[\mathcal{F}^{CONF}\right] \subsetneq [\mathcal{F}]$.

Lemma 3. $\mathcal{L} \in \left[\mathcal{F}^{CONF}\right]$.

Lemma 4. *Let* $\varphi \in \mathcal{F}^{CONF}$*. Then for any* $L \in RE$ *there is a* $\sigma \in SEQ$ *so that*

(1) $rng(\sigma) \subset \mathcal{L}$*, and*
(2) $\forall \tau \in SEQ\ rng(\tau) \subset \mathcal{L} \implies \varphi(\sigma\hat{}\tau) = \varphi(\sigma)$.

Proposition 2. $\left[\mathcal{F}^{REC} \cap \mathcal{F}^{CONF}\right] \subsetneq \left[\mathcal{F}^{REC}\right] \cap \left[\mathcal{F}^{CONF}\right]$.